THE TRANSFER OF HEAT BETWEEN MOVING BODIES IN CONTACT

N. V. Diligenskii and V. V. Basov

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We present a solution for the problem of the distribution of temperature and that of the intensity of heat flows in bodies that are in contact in the case of steady-state heat transfer and great velocities of motion.

The description of certain technological processes is frequently reduced to the following heat problem (Fig. 1).

A rod II is moving at a velocity v_1 in the direction of the x-axis over the surface of a semiinfinite plate I. The rod is moving simultaneously at a velocity $v_2 = v_1/k$ in the plane perpendicular to the vector of the velocity v_1.

At each point of contact between the rod and the semiinfinite plate a quantity of heat equal to q is liberated. The initial temperature of the bodies is t_0.

We have to find the temperature distribution at the contact area and to determine the quantity of heat which is transferred to each of the bodies in contact.

Let us examine the solution of this problem for the case of steady-state heat transfer and for the case of great velocities of body motion.

We will assume the thermophysical properties of the bodies to be constant and identical, as well as independent of temperature.

We will assume the bodies in contact to be thin, and we will neglect the variation in temperature through the body thickness.

Let us write [1] the equation for the propagation of heat in a semiinfinite plate in a coordinate system associated with the heat source in the following manner:

$$v_1 \frac{\partial t}{\partial x} = a\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2}\right),$$

or in dimensionless form

$$\mathrm{Pe}\frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2}. \tag{1}$$

At a great velocity, the temperature gradient in the direction ξ of the displacement of the heat source is small in comparison with the gradient along the normal to the displacement axis, i.e.,

$$\frac{\partial \theta}{\partial \xi} \ll \frac{\partial \theta}{\partial \eta}.$$

This provides a basis for replacing the process of heat propagation in the plate by a set of independent heat-propagation processes in elementary rods perpendicular to the x-axis [2].

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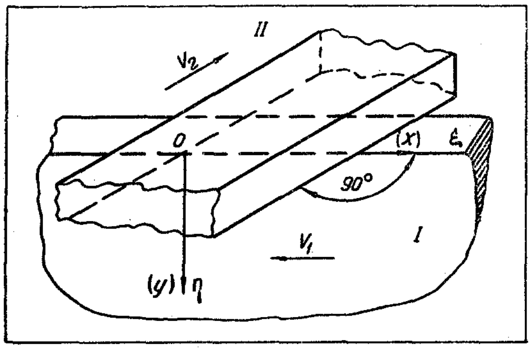

Fig. 1. Diagram for the calculation of the transfer of heat between two bodies in contact (I and II are semiinfinite plates).

Equation (1) then assumes the form

$$\mathrm{Pe}\frac{\partial\theta}{\partial\xi}=\frac{\partial^2\theta}{\partial\eta^2}\,. \tag{2}$$

In practical terms, Eq. (2) can be used to calculate the temperatures of the contact and near-contact regions for the case in which Pe > 10 [3].

In thermophysics it is frequently necessary to deal with high-speed processes in which the Pe numbers are in the order of hundreds, thousands, and more, and the assumption that it is possible to neglect the flow of conduction heat along the x-axis introduces no perceptible error into the engineering calculations.

The boundary conditions for (2) are

$$\theta|_{\xi=0}=0, \tag{3}$$

$$\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta\to\infty}=0. \tag{4}$$

We determine the boundary condition for $\eta = 0$ from the condition of the heat balance at the contact area.

The direction of the heat flow q_r to the rod, in the case of a great rod velocity v_2, virtually coincides with the direction of motion and q_r is thus linearly dependent on the contact temperature:

$$q_r=c_p\gamma v_2(t_c-t_0).$$

Consequently, the heat-balance equation for each of the points on the contact surface is written in the following manner:

$$-\lambda\frac{\partial t}{\partial n}=q-c_p\gamma v_2(t_c-t_0)$$

or in dimensionless form

$$-\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0}=\mathrm{Ki}-\frac{\mathrm{Pe}}{k}\theta_c\,. \tag{5}$$

We note that the fraction of heat reaching the rod from each of the contact points, in relative terms, amounts to

$$\omega_r=\frac{\mathrm{Pe}}{\mathrm{Ki}\,k}\theta_c, \tag{6}$$

while in the half-plane we have

$$\omega_h=1-\omega_r\,. \tag{7}$$

Expression (5) is the boundary condition for (2) when $\eta = 0$ and $\xi \in [0, 1]$.

Applying the Laplace transform to (2) with conditions (3), (4), and (5) in the domain $\xi \in [0, 1]$, we obtain

$$\theta=\frac{\mathrm{Ki}\,k}{\mathrm{Pe}}\left[\operatorname{erfc}\left(\frac{\eta\sqrt{\mathrm{Pe}}}{2\sqrt{\xi}}\right)-\exp\left(\frac{\mathrm{Pe}}{k}\eta+\frac{\mathrm{Pe}}{k^2}\xi\right)\operatorname{erfc}\left(\frac{\sqrt{\mathrm{Pe}}}{2\sqrt{\xi}}\eta+\frac{\sqrt{\mathrm{Pe}}}{k}\sqrt{\xi}\right)\right]. \tag{8}$$

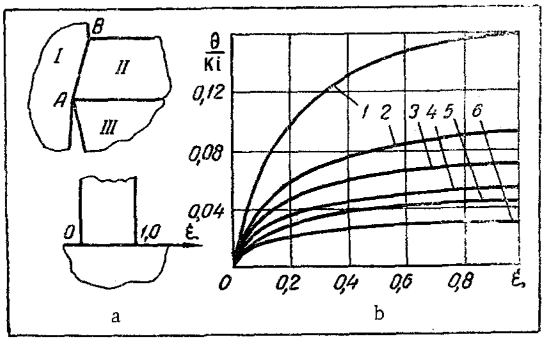

Fig. 2

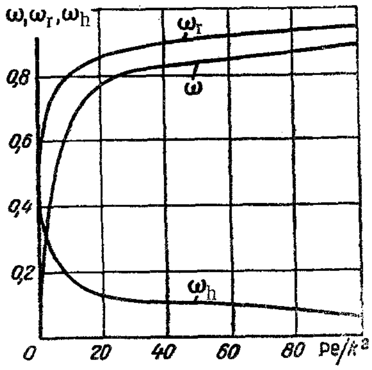

Fig. 3

Fig. 2. Diagram of free cutting: I) material; II) cutter; III) shavings; A-B) strain zone; a) distribution of temperature along the length of the shaving strain zone as a function of cutting speed; [1) Pe = 10; 2) 20; 3) 30; 4) 40; 5) 50; 6) 100]; b) k = 2.5.

Fig. 3. Heat balance for the cutting as a function of the cutting speed.

We can use expression (8) to determine the temperatures in the near-contact region I of the body.

Assuming in (9) that $\eta = 0$, we write the formula for the determination of the temperature in the contact area:

$$\theta_c = \frac{\mathrm{Ki}\, k}{\mathrm{Pe}} \left[1 - \exp\left(\frac{\mathrm{Pe}\,\xi}{k^2}\right) \operatorname{erfc}\left(\frac{\sqrt{\mathrm{Pe}\,\xi}}{k}\right)\right]. \tag{9}$$

Bearing in mind (6) and (7), we find the heat-balance data

$$\omega_r = 1 - \exp\left(\frac{\mathrm{Pe}\,\xi}{k^2}\right) \operatorname{erfc}\left(\frac{\sqrt{\mathrm{Pe}\,\xi}}{k}\right), \tag{10}$$

$$\omega_h = \exp\left(\frac{\mathrm{Pe}\,\xi}{k^2}\right) \operatorname{erfc}\left(\frac{\sqrt{\mathrm{Pe}\,\xi}}{k}\right). \tag{11}$$

Figure 2a shows an idealized diagram of free cutting. Shavings are removed in the strain zone A-B and the heat of deformation q is released. In this diagram, the transfer of heat between object I and shavings II can be treated in accordance with the above-cited theoretical scheme [3]. In this case v_1 is the cutting speed, and k is the coefficient of shavings shrinkage. Figure 2b shows the curves for the change in temperature over the length of the strain focus for various cutting speeds.

An examination of Fig. 2 shows that at high cutting speeds the temperature in the strain zone varies insignificantly, with the exception of the small initial segments in which there is a pronounced variation in the initial temperature, subsequently becoming asymptotic. The fact of the constancy of the strain temperature agrees with the data of other researchers [3].

The profile for the intensity of the heat flow removed with the shavings, in the light of (6), coincides with the temperature profile and also exhibits a maximum toward the end of the strain zone.

Having integrated (8) with respect to ξ from 0 to 1, we obtain an expression for the mean integral strain temperature

$$\theta_{st} = \frac{\mathrm{Ki}\, k}{\mathrm{Pe}}\, \omega, \tag{12}$$

where

$$\omega = 1 - \frac{2k}{\sqrt{\pi\,\mathrm{Pe}}} - \frac{k^2}{\mathrm{Pe}} \exp\left(\frac{\mathrm{Pe}}{k^2}\right) \operatorname{erfc}\left(\frac{\sqrt{\mathrm{Pe}}}{k}\right). \tag{13}$$

Bearing in mind (6), we see that ω is the ratio of the quantity of heat removed by the shavings to all of the heat that has been released.

Figure 3 shows the curves for the change in ω, ω_r, ω_h as a function of the complex Pe/k^2 for $\xi = 1$.

From analysis of Fig. 3 we can draw the conclusion that at high cutting speeds the values of the average and maximum intensities of heat flows to the shavings (or the average and maximum strain temperatures) differ little from each other; the principal fraction of the liberated heat is removed together with the shavings.

Let us note that for $Pe\,\xi/k^2 > 10$ formulas (9)-(11) are substantially simplified if we use the asymptotic representation of the erfc function

$$\theta_c \simeq \frac{Ki\,k}{Pe}\left(1 - \frac{k}{\sqrt{\pi\,Pe\,\xi}}\right), \tag{14}$$

$$\omega_r \simeq 1 - \frac{k}{\sqrt{\pi\,Pe\,\xi}}, \tag{15}$$

$$\omega_h \simeq \frac{k}{\sqrt{\pi\,Pe\,\xi}}. \tag{16}$$

When $Pe/k^2 > 10$, formula (13) is simplified to

$$\omega = 1 - \frac{2k}{\sqrt{\pi\,Pe}}. \tag{17}$$

In conclusion, we note that in [5] equations and boundary conditions similar to (2), (3), and (5) were examined. However, the physical formulation of the problem and the method of solution differ from those adopted in the cited reference.

NOTATION

v_1	is the velocity at which body I is moving in the direction of the x-axis;
v_2	is the velocity of body II;
k	is the shrinkage factor;
x and y	are coordinates associated with the rod;
t	is the temperature;
a	is the coefficient of thermal diffusivity;
$Pe = vl/a$	is the Peclet number;
ξ and η	are dimensionless coordinates, $\xi = x/l$, $\eta = y/l$;
l	is the contact length;
θ	is the dimensionless temperature, $\theta = (t - t_0)/(t_{sc} - t_0)$
t_{sc}	is the scale temperature;
t_0	is the initial temperature;
q_r	is the specific quantity of heat transferred to rod I;
c_p	is the specific heat capacity;
γ	is the density;
λ	is the coefficient of thermal conductivity for body I;
Ki	is the Kirpichev number;
ω_r	is the ratio of the quantity of heat to the rod at each point of contact to the quantity of heat liberated at that point;
ω_h	is the same, with reference to the half-plane;
θ_s	is the average relative temperature at the contact area;
ω	is the average quantity of heat to the rod relative to the overall heat liberated at the contact area.

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